

Please show your work, and your thinking, in the space provided. Be brief, but complete. Long, wandering answers typically demonstrate a lack of understanding...

$$\hbar = 1.054 \times 10^{-34} \text{ J s}$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$h = 6.626 \times 10^{-34} \text{ J s}$$

$$4\pi\epsilon_0 = 1.113 \times 10^{-10} \text{ J}^{-1} \text{ C}^2 \text{ m}^{-1}$$

$$m_e = 9.109 \times 10^{-31} \text{ kg}$$

$$N_0 = 6.022 \times 10^{23} \text{ mol}^{-1}$$

$$c = 2.998 \times 10^8 \text{ m s}^{-1}$$

$$\pi = 3.14159$$

$$k = 1.381 \times 10^{-23} \text{ J K}^{-1}$$

$$H_{\text{translational}} = \frac{-\hbar^2}{2m} \frac{d^2}{dx^2}$$

$$P_{\text{translational}} = \frac{\hbar}{i} \frac{d}{dx}$$

**Particle in a 1D box**

$$\psi_n = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{n\pi x}{L}\right)$$

$$E_n = \frac{n^2 \hbar^2}{8mL^2} \quad n = 1, 2, 3, \dots$$

**1D Harmonic oscillator**

$$\psi_v = N_v H_v(y) e^{-\frac{1}{2}y^2} \quad y = \frac{x}{\alpha} \quad \alpha = \left(\frac{\hbar^2}{mk}\right)^{1/4}$$

$$N_v = \left(\frac{1}{2^v v! \pi^{1/2} \alpha}\right)^{1/2} \quad H_v(y) = \text{Hermite Polyn}$$

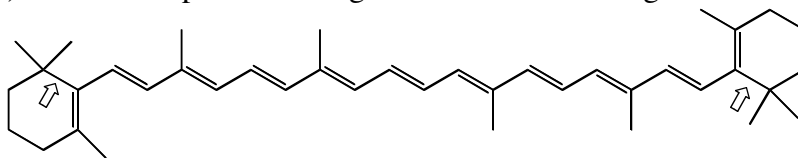
$$E_v = \left(v + \frac{1}{2}\right) \hbar\omega \quad v = 0, 1, 2, 3, \dots$$

1. Consider the wavefunction:  $\psi = k\hbar e^{ikx}$

a) (20 points) Show that it is an eigenfunction of H

b) (10 points) What can you conclude about the *motion* of an electron described by this wavefunction? Your conclusions should be both quantitative and qualitative.

2. (20 points) The chromophore which gives carrots their orange color is beta-carotene.



The color of tomatoes is due to a similar molecule, in which the rings have undergone scission where indicated. What color are tomatoes (*explain* briefly)?

3. (25 points) An electron is confined to a one dimensional box of length  $L$ . What should the length of the box be in order for the electron's zero point energy to be equal to its rest mass energy of  $m_e c^2$ ?

4. a) (15 points) The potentials below left show the scenario described for the simple one-dimensional particle in a box. Draw on top of the potentials, the functions for the  $n=1$  (top) wavefunction and the  $n=2$  (lower) wavefunction.

b) (10 points) The potentials below right show a slightly modified potential in which one “wall of the box” is not at infinitely high potential. Draw the same wavefunctions for  $n=1$  and  $n=2$ , but illustrating how the wavefunction will change in this new potential. Think about what you expect from the limits of classical behavior.

